Analysis of Actuator In-phase Property in Terms of Control Performance and Integrated Plant/Controller Design Using a Novel Model Matching Method

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Abstract—This paper is concerned with resonance in-phase property of a VCM (voice coil motor) plant system in the sense of control performance in HDDs (hard disk drives). Its relationships with the optimal performance level $\gamma_{opt}$, the stability margins and the disturbance rejection capability are revealed. It is found that the main resonance being in-phase is particularly beneficial to rejection of the narrow-band disturbances with frequencies near plant resonances. In order to meet the requirement on the in-phase property, a partial model matching method is proposed. This model matching problem is solved by an $H_{\infty}$ method using an linear matrix inequality approach. The partial model matching method is then applied to the VCM plant system. We especially take into account the in-phase case for the purpose to improve the system ability to attenuate high frequency disturbance. For the new system designed using the proposed model matching method, a feedback controller and a group peak filter are designed to attenuate the disturbance near the plant resonances. The advantages of the in-phase resonances are illustrated, when compared with the original plant.

I. INTRODUCTION

System properties such as in-phase (IP), positive-realness (PR) and minimum-phase (MP) are known to be crucial to achievable control performance. How they are related to the ultimately achievable control performance is discussed in detail in [1], [2], [3]. Particularly, it is claimed that good control performance can be expected if a mechanical system is designed such that the resulting transfer function has in-phase property [2], [3]. Devoted to a VCM actuator based mechanical system in HDDs, reference [4] presents an actuator design concept for which higher-order resonances with high gain are designed in-phase to the first major resonance and out-of-phase resonances are designed with reduced gains.

At present, a main development trend in hard disk drives is the ever-increasing storage capacity and track density, which demands a high-level performance servo system that allows the read/write head to accurately position along the center of a track. This kind of servo control systems, in order to reach nano-meter level positioning accuracy, must be able to effectively attenuate disturbances caused by air flow, disk vibration, spindle runout and actuator resonance modes. Disturbances such as air-flow induced vibrations [5] and head disk interface induced vibrations [6] that appear to be narrowbanded and occur at some particular frequencies become major track misregistration (TMR) sources. These kinds of narrowband disturbances near the frequency of a few kHz are critical to improving the positioning accuracy of the read/write head.

Conventionally, the narrowband disturbances are rejected with a peak filter [7], [8], [9]. Recently, blending control technique is applied to simultaneously reject two or three narrow-band disturbances at a few kHz [10]. However, for the disturbances near plant resonance modes, these methods become less useful, especially for high frequency disturbances above 10 kHz.

Considering the importance of the plant resonance property to control performance, we would like to study connection of the in-phase property of resonances and the high frequency disturbance rejection.

Because of the known relationship between the plant properties and the control performance, integrated plant/control design methodology has attained much attention recently [11], [12]. The integrated design includes two steps and starts from the plant modification so that it possesses some beneficial properties, which is followed by standard optimal control methods that are applied to complete the whole control system design. [13] considers an integrated design for hard disk drive servo systems, where servo bandwidth limitation is analyzed in relation to the primary resonance frequency and its residue and a new mechanical system is proposed. In this paper, we utilize model matching concept to change the properties of the plant. We are specifically concerned with in-phase resonances. The model matching problem consists of designing a controller to compensate a given plant so that the resultant controlled system approximately matches a prespecified reference model using some distance measure [14], [15], [16]. Here we adopt the $H_{\infty}$ norm of the error between the two models corresponding to any plant input as the distance measure. The reference model transfer function can be given to encompass all performance requirements in both frequency and time domains. In this paper, the reference model is produced with the required in-phase resonances contained.

This paper focuses on the VCM plant system in HDDs and intends to show some relationship between the plant in-phase property and some achievable performance by a certain optimal control. The optimal performance level $\gamma_{opt}$ in $H_{\infty}$ loop shaping design [1] is adopted to evaluate the in-phase property.
of the plant flexible resonances. Control performances such as stability margins and error rejection capability are analyzed for different in-phase resonances. Based on the analysis, we especially look into the case of the main resonance being in-phase for the purpose to improve the system ability to suppress high frequency disturbances. In order to meet the requirement on the plant, we propose a partial model matching method to modify the original plant. The partial model matching can give a resultant modified system of lower order, save computational resources and speed up the design procedure, compared with the conventional model matching for entire plant model. The model matching is implemented by utilizing an $H_\infty$ method and solved by a linear matrix inequality approach. Based on the newly designed system, we proceed to design a normal feedback controller as well as a group peak filter to attenuate disturbances near the resonances. The advantages of the in-phase resonances due to the model matching are illustrated, when compared with the original plant, in terms of the performance in high frequency disturbance rejection.

II. RELATIONSHIP ANALYSIS OF IN-PHASE RESONANCES AND CONTROL PERFORMANCE

In this section, we reveal how the in-phase property of a VCM plant in HDDs relates to control performance such as stability margins, bandwidth, and error rejection capability, especially to disturbance with frequencies at resonance modes.

The model of the VCM plant in HDDs is generally written as

$$P_e(s) = K \frac{1}{s^n} + \sum_{i=1}^{n} \frac{K_i}{s^i + 2\xi_i\omega_i s + \omega_i^2}$$

(1)

with the rigid part $K$ and $n$ resonances at frequencies $\omega_i = 2\pi f_i$ ($i = 1, 2, 3, \cdots, n$). When there is a time delay term involved in the plant system and described by

$$P_d(s) = \frac{-Ts + 1}{Ts + 1},$$

(2)

the complete plant model is then written as

$$P(s) = P_d(s)P_e(s).$$

(3)

The resonance mode $i$ is said to be in-phase if the coefficient $K_i$ has the same sign as $K$ [2]. Otherwise, it is said to be out-of-phase.

A. Relationship with $\gamma_{\text{opt}}$

It is known that good control performance can be expected if a mechanical system is designed such that the resulting transfer function has an in-phase property. In this section, we show some relationship between the in-phase property and the optimal performance level $\gamma_{\text{opt}}$ in $H_\infty$ loop shaping design for the VCM plant system.

In the feedback control loop in Fig. 1 with $A/D$ and $D/A$ converters for controller implementation, $P$ is the given plant to be controlled, and $d$ is the disturbance with the contribution on the error $e_c$ by $e_c = S(z)d$ through the sensitivity function

$$S(z) = \frac{1}{1 + P(z)c(z)}.$$
satisfied, which means that the sensitivity function \( S(z) \) can be shaped similarly to the inverse of the chosen weighting function \( W(z) \). A simple form of \( W(s) \) is chosen as

\[
W(s) = \frac{s^2 + 2\xi \omega s + \omega^2}{s^2 + 2\omega \sqrt{\varepsilon} s + \omega^2 \varepsilon}
\]  

(8)

where \( \omega \) is valued by the desired bandwidth, \( \varepsilon \) is to determine the low-frequency level of the desired sensitivity function, and \( \xi \) is the damping ratio. \( W(z) \) is the discretized form of \( W(s) \). More details about how to obtain the feedback controller \( C(z) \) are described in [18].

Here, we select \( \omega = 2\pi900 \), \( \varepsilon = 10^{-4.5} \), and \( \xi = 1 \). The sampling rate is 40 kHz. The resultant gain/phase margins as well as bandwidth are listed in Table II. Better control performances such as higher gain/phase margins and higher bandwidth are expected with more in-phase resonances or lower \( \gamma_{opt} \). Differences of the closed-loop sensitivity functions are observed clearly in Fig. 2, and when all resonances are in-phase, the best one with highest bandwidth and lowest hump is achieved.

### TABLE II

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>2+</th>
<th>3+</th>
<th>3+4+</th>
<th>2+3+4+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain margin (dB)</td>
<td>9.8</td>
<td>10.7</td>
<td>9.9</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Phase margin (deg)</td>
<td>50</td>
<td>51.7</td>
<td>50</td>
<td>54</td>
<td>63</td>
</tr>
<tr>
<td>Bandwidth (Hz)</td>
<td>1.4</td>
<td>1.5</td>
<td>1.4</td>
<td>1.56</td>
<td>1.85</td>
</tr>
</tbody>
</table>

More details about how to obtain the feedback controller \( C(z) \) are described in [18], [9]. To attenuate the disturbance contribution to the error, which works together with \( C(z) \) as \( C(z)[1 + F(z)] \).

The group peak filter adopts the following form

\[
F(s) = \sum_{i=1}^{\nu} F^i(s),
\]

(9)

where the sub-filters \( F^i(s) \) are given by

\[
F^i(s) = L_i - \frac{\sin(\phi_i)s^2 + \omega_i \cos(\phi_i)s}{s^2 + 2\xi_i \omega_i s + \omega_i^2},
\]

(10)

with the corresponding filter parameters \( (\omega_i, \phi_i, \xi_i, L_i) \). \( \omega_i \) is the frequency at which the disturbance is to be rejected. \( \xi_i \in (0, 1) \) is the damping ratio. \( \phi_i \), and \( L_i \) are determined via baseline control system with \( C(s) \) and \( P(s) \). The details about the peak filter design are seen in [8], [9].

\( F(z) \) is the discrete form of \( F(s) \). Notice that the main resonance of the plant is at 9.7 kHz. We find that with the main resonance being in-phase, the best performance for the disturbance rejection is obtained. Fig. 3 shows its comparison with the original plant. The disturbance rejection is much better and no side-effect or no additional upside peak appears near 9.7 and 14 kHz. Also the system stability is not destroyed.

We should mention that the case with all resonances being in-phase, which gives the best performance as in the above section, cannot continue to perform the best in terms of this high frequency disturbance rejection.

![Fig. 2. Comparison of sensitivity functions.](image)

### C. Effect on high frequency disturbance rejection

Assume that the disturbance \( d \) is composed of three narrow-band components with center frequencies at the plant resonance frequencies: 7.3, 9.7 and 14 kHz. Rejection to the disturbance with such a high frequency is rather difficult as the control system stability is easily affected and some side effect may be caused such as amplification of other disturbances. In addition to the controller \( C(z) \), we design a group peak filter \( F(z) \) [8], [9] to attenuate the disturbance contribution to the error, which works together with \( C(z) \) as \( C(z)[1 + F(z)] \).

In view of the above analysis, the integrative plant/controller design will be applied that is to change the original system such that the new system has desired properties such as in-phase resonances, and subsequently design a feedback controller for the new system to realize the desired performance. In the next section, we propose a model matching method to design the new system.

### III. A PROPOSED MODEL MATCHING METHOD

Conventionally, model matching is conducted for the entire system \( P_o(s) \). However, because usually some part of the
system is not necessarily changed and only the other part of the system is to be modified, the model matching is just needed for part of the system. Compared with the conventional model matching, this partial model matching can give a resultant modified system of lower order, save computational resources and speed up the design procedure.

Partition $P_v(s)$ in (1) as follows.

$$P_v(s) = \tilde{P}(s) + \hat{P}(s),$$  \hspace{1cm} (11)

where $\tilde{P}(s)$ is a fixed part, and $\hat{P}(s)$ is a part to be changed such as some out-of-phase resonance modes. As plotted in Fig. 4, $\Gamma(s)$ is a given target model. The aim is to design $M(s)$ such that the error $e(t) = y(t) - \hat{y}(t)$ is minimized for any $u_p$, and thus the new model $P_m(s) \approx \tilde{P}(s) + \Gamma(s)$. If $\tilde{P}(s)$ is composed of out-of-phase resonance modes, the target model $\Gamma(s) = -\hat{P}(s)$ is of in-phase. The overall target model is

$$P_t(s) = \tilde{P}(s) + \Gamma(s) = \hat{P}(s) - \hat{P}(s).$$  \hspace{1cm} (12)

If the system $P_v(s)$ is invertible, $\frac{\hat{P}(s)}{P_v(s)}$ can be used as a compensator cascaded with $P_v(s)$ straightforwardly. But generally for most of systems, exact inverse does not exist. Therefore, we propose a model matching method to design a $M(s)$.

![Fig. 4. Partial model matching.](image)

Denote the state-space realizations: $\hat{P}(s)$: $(A_{\hat{P}}, B_{\hat{P}}, C_{\hat{P}}, D_{\hat{P}})$ with state $x_{\hat{P}}$; $\tilde{P}(s)$: $(A_{\tilde{P}}, B_{\tilde{P}}, C_{\tilde{P}}, D_{\tilde{P}})$ with state $x_{\tilde{P}}$; $\Gamma(s)$: $(A_{\Gamma}, B_{\Gamma}, C_{\Gamma}, D_{\Gamma})$ with state $x_{\Gamma}$. A combined system is derived from Fig. 4 and given by

$$\begin{bmatrix} \dot{x}_{\hat{P}}(t) \\ \dot{x}_{\tilde{P}}(t) \\ \dot{x}_{\Gamma}(t) \end{bmatrix} = \begin{bmatrix} A_{\hat{P}} & 0 & 0 \\ 0 & A_{\tilde{P}} & 0 \\ 0 & 0 & A_{\Gamma} \end{bmatrix} \begin{bmatrix} x_{\hat{P}}(t) \\ x_{\tilde{P}}(t) \\ x_{\Gamma}(t) \end{bmatrix} + \begin{bmatrix} B_{\hat{P}} \\ B_{\tilde{P}} \\ B_{\Gamma} \end{bmatrix} u_p(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u_m(t),$$  \hspace{1cm} (13)

$$y(t) = \begin{bmatrix} C_{\hat{P}} & C_{\tilde{P}} & 0 \end{bmatrix} \begin{bmatrix} x_{\hat{P}}(t) \\ x_{\tilde{P}}(t) \\ x_{\Gamma}(t) \end{bmatrix} + (D_{\hat{P}} + D_{\tilde{P}}) u_p(t) + D_{\tilde{P}} u_m(t).$$  \hspace{1cm} (14)

$$e(t) = \begin{bmatrix} 0 & C_{\Gamma} \end{bmatrix} \begin{bmatrix} x_{\hat{P}}(t) \\ x_{\tilde{P}}(t) \\ x_{\Gamma}(t) \end{bmatrix} + (D_{\hat{P}} - D_{\Gamma}) u_p(t) + D_{\tilde{P}} u_m(t).$$  \hspace{1cm} (15)

With this combined system, the $M(s)$ design problem is thus converted to a typical $H_{\infty}$ control method by minimizing the $H_{\infty}$ norm of the transfer function from $u_p$ to the error $e$. Approaches are available to design $M(s)$, such as a linear matrix inequality (LMI) approach and an $H_{\infty}$ control function in Matlab.

Note that the minimization of $||\hat{y} - y||_\infty$ can guarantee $\tilde{y}$ approaches $\hat{y}$ for any energy bounded $u_p$, which means that

$$P_m(s) = \frac{\hat{y} + \tilde{y}}{u_p} \approx \frac{\tilde{y} + y}{u_p} = \tilde{P}(s) + \Gamma(s),$$  \hspace{1cm} (16)

i.e.,

$$P_m(s) \approx P_t(s) = \hat{P}(s) - \hat{P}(s).$$  \hspace{1cm} (17)

In this sense, the proposed partial model matching method applies to any target model $\Gamma(s)$, as long as the minimization problem of $H_{\infty}$ norm $||\hat{y} - y||_\infty$ is solvable. Given by the plant $P_m(s)$ modified by the target model, feedback controls can be designed to achieve the required control performance based on the target model.

Further derivation finds that the modified plant $P_m(s)$ in Fig. 4 equals to

$$P_m(s) = \frac{y}{u_p} = \frac{\hat{P}(s) + \tilde{P}(s)}{1 - M(s)P(s)}.\hspace{1cm} (18)$$

In the next section, we conduct the partial model matching on the VCM plant in HDDs to change specific out-of-phase resonances to be in-phase and look into the effect of the in-phase resonance modes on the rejection capability of the resultant control system to high frequency disturbances.

IV. HIGH FREQUENCY DISTURBANCE REJECTION IN HDD SERVO SYSTEMS

According to the analysis in Section II.C, the plant main resonance at 9.7 kHz being in-phase is most beneficial to the performance of disturbance rejection for disturbances at 7.3, 9.7, and 14 kHz. Thus we would like to use the partial model matching method to design a new system with the resonance at 9.7 kHz being in-phase.

A. Partial model matching

We now use the partial model matching method proposed previously to change the 2nd resonance mode at 9.7 kHz to be in-phase modes. $\hat{P}$ and $P$ are then given by

$$\hat{P}(s) = \frac{K_2}{s^2 + 2\xi_2\omega_2s + \omega_2^2},$$  \hspace{1cm} (19)

$$P(s) = \frac{K}{s^2 + 2\xi_1\omega_1s + \omega_1^2} + \sum_{i=3}^{4} \frac{K_i}{s^2 + 2\xi_i\omega_is + \omega_i^2}.$$  \hspace{1cm} (20)

The target $\Gamma(s) = -\hat{P}(s)$. 
Using an LMI approach to solve the $H_\infty$ problem associated with the combined system (13)-(15), $M(s)$ can be obtained with the minimized $H_\infty$ norm $8e^{-5}$. The comparison between the resultant modified plant $P_m(s)$ and the target plant $P_t(s) = \hat{P}(s) + \Gamma(s)$ is shown in Fig. 5 in terms of frequency responses.

$$P_m(z) = \left[1 + F(z)\right]C(z) \frac{1}{1 - M(z)\hat{P}(z)} \tag{21}$$

where $M(z)$ and $\hat{P}(z)$ are the discretized forms of $M(s)$ and $\hat{P}(s)$ with 40 kHz sampling rate. Fig. 7 shows the frequency response of the control $\Sigma(z)$. The open-loop frequency responses are compared in Fig. 8 with the ideal case in Section II.C. The gain/phase margin and bandwidth are 8.6 dB, 47.8 deg, and 1.45 kHz, which are comparable with 10.7 dB, 51.7 deg, and 1.5 kHz for the ideal case in Section II.C.

The implementation is done with a laser Doppler vibrometer (LDV) and a dSPACE 1103. The dSPACE is used to implement the controller $\Sigma(z)$ and the LDV is used to measure the VCM actuator position of the hard disk drive. The open-loop $P(z)\Sigma(z)$ frequency responses and the sensitivity function $S(z) = 1/[1 + P(z)\Sigma(z)]$ of the closed-loop are measured via a Dynamic Signal Analyzer (DSA) and shown in Figs. 9-10. Fig. 9 agrees well with Fig. 8, and Fig. 10 is very close to the ideal case in Fig. 3.

**V. Conclusion**

In-phase property of the VCM plant system in HDDs has been considered and analyzed in terms of control performances in this paper. It has been shown that in-phase property has led to lower performance level $\gamma_{opt}$, and better control performance such as better stability margins and disturbance rejection capability. It has been found that the main resonance...
being in-phase is particularly beneficial to the rejection of the narrow-band disturbances with the frequencies near plant resonances.

To generate a new plant model with certain in-phase resonances, a partial model matching method has been proposed and an H∞ method together with a linear matrix inequality approach has been used to solve the model matching problem. Compared with the conventional model matching for the entire plant model, the partial model matching can give a resultant modified system of lower order, save computational resources and speed up the design procedure. It also applies to other new plant with required properties other than the in-phase property.

To show the goodness of the new plant with the desired in-phase property, the feedback controller and the group peak filter have been designed for it. Then, the advantages of the in-phase resonances in the disturbance rejection with the frequencies near plant resonances have been illustrated, which agree quite well with the previous analysis on in-phase resonances.

REFERENCES