Adaptive Feedback Control of Magnetic Suspension System preceded by Bouc-Wen Hysteresis

Jing Zhou
Faculty of Engineering and Industrial Sciences
Swinburne University of Technology
Hawthorn, 3122 Victoria, Australia
Email: jingzhou@swin.edu.au

Changyun Wen
School of Electrical and Electronic Engineering
Nanyang Technological University
Nanyang Avenue, 639798, Singapore
Email: ecywen@ntu.edu.sg

Abstract—In this paper, we consider a class of uncertain magnetic suspension system preceded by Bouc-Wen type of hysteresis nonlinearity. A new perfect inverse function of the hysteresis is constructed and used to cancel the hysteresis effects in controller design with backstepping technique. For the design and implementation of the controller, no knowledge is assumed on system parameters. It is shown that the proposed controller not only guarantees asymptotic stability, but also transient performance.

Keywords: Adaptive control, backstepping, hysteresis, magnetic suspension system, inverse

I. INTRODUCTION

Hysteresis exists in a wide range of physical systems and devices, such as biology optics, electro-magnetism, mechanical actuators, electronic relay circuits and other areas. Such nonlinearity often limits system performance. Control of systems with hysteresis nonlinearity is an important area of control system research and typically challenging. To address such a challenging, it is important to find a model which can describe the nonlinear behavior of the hysteresis process and be utilized for control design. Several mathematical models have been proposed for this purpose, such as Duhem model [1], Preisach model [2], Krasnosel’ski-Pokrovkii hysteron [3], Prandtl-Ishlinskii hysteresis operator [1], Bouc-Wen differential model [4], [5], [6], [7], and lines-segments hysteresis model in [8].

Research on adaptive control of systems with hysteresis nonlinearity has received great attention in recent years. In [9], the stability of systems with hysteresis was investigated, where a passive hysteresis operator was developed for Preisach hysteresis model. For lines-segments hysteresis nonlinearity, the inverse is easily obtained, see for examples [8], [10], [11] and [12]. In [11] and [12], a non-smooth inverse nonlinearity model was constructed. In [13], an efficient smooth adaptive inverse was developed to compensate for the effect of the backlash in controller design with backstepping approach. When the hysteresis is represented by differential equations, the inverse is either impossible or extremely difficult to be obtained. In [14] and [15], adaptive control approaches were developed to deal with such type of backlash-hysteresis. Rather than constructing an inverse model, the effect of backlash was treated as a bounded disturbance and the detailed characteristic of backlash-like hysteresis was not considered in the above papers. In [16], another adaptive control approach was fused with the Prandtl-Ishlinskii model without constructing a hysteresis inverse, since the inverse is also difficult to be obtained. It is noted that the control methods in [14], [15], [16] may result in large control input magnitudes. In [17], approximate inversion of the Preisach hysteresis operator with application to control of smart actuators was studied. An approximate implicit inverse approach was introduced for linear discrete-time dynamical systems preceded with Prandtl-Ishlinskii hysteresis in [18]. The approximate implicit inversion was obtained by searching for an optimal value of the inversion based on a performance index. However, this approximate implicit inverse is dependent on the preceded system and only illustrated for discrete-time linear systems. The above concept of constructing an implicit inversion is revised for continuous-time linear systems with Prandtl-Ishlinskii hysteresis in [19], where a perfect implicit inversion of Prandtl-Ishlinskii hysteresis was developed without approximation and was associated with an adaptive control scheme.

It is noted that the Bouc-Wen differential model in [4], [5], [6] is one of the most widely accepted phenomenological models of the hysteresis in mechanics. Actually it can be shown that the hysteresis model presented in [14], [15] is a special case of the Bouc-Wen hysteresis model. So far, controlling systems with the Bouc-Wen hysteresis has only been reported in the works of [20],[21], and [22]. In [20], an adaptive controller was designed for a second-order structural system with unknown hysteresis to counteract the effect of an earthquake excitation, where the system uncertain parameters must be within some known intervals and the effect of the hysteresis is treated in a similar way to that of handling a bounded disturbance. In [21], adaptive backstepping control was developed for the same class of systems in [20], where the structural parameters and isolation parameters are all uncertain parameters. The detailed characteristic of hysteresis was not considered in the controller design. In [22], an inverse multiplicative structure was proposed to compensate for hysteresis nonlinearity in piezoelectric materials. Since the Bouc-Wen hysteresis is generated from differential equations, the development of hysteresis inverse is difficult. In [23], a simple output feedback adaptive control scheme for a general class of nonlinear systems preceded by an actuator with hysteresis described by a modified Bouc-Wen...
differential model, where the hysteresis is taken into account in controller design, instead of only considering its effect like bounded disturbances as in [14], [15], [20], [21].

In this paper, we develop a simple feedback adaptive control scheme for a magnetic suspension system preceded by an actuator with hysteresis described by a modified Bouc-Wen differential model. To improve system performance, we take the hysteresis into account in controller design. To achieve this, a new hysteresis inverse is obtained for the hysteresis and is used to efficiently cancel the hysteresis effects when developing the control scheme with backstepping approach. In our design, no knowledge is assumed on system parameters. Besides showing the stability of the close-loop system, output tracking error is also ensured asymptotically approach to zero. In addition, transient performance in terms of $L_2$ norm of the tracking error is derived to be an explicit function of design parameters.

II. BOUC-WEN HYSTERESIS CHARACTERISTIC

The extended Bouc-Wen differential model in [4], [5], [6] is one of the most widely accepted phenomenological models of the hysteresis in mechanics. It was proved in [7] that the parameters of the Bouc-Wen model in [4], [5] are functionally redundant. That is, there exist multiple parameter vectors that are related to the pseudo-natural frequency of the nonlinearity, $\mu$, and $\kappa$ is a parameter related to the smoothness of the transition from initial slope to the slope of the asymptote. $f(\zeta, \dot{v}) \in \mathbb{R}^1$ is a function with two inputs $\zeta \in \mathbb{R}^1$ and $\dot{v} \in \mathbb{R}^1$ defined as

$$f(\zeta, \dot{v}) = 1 - \text{sign}(\dot{v})|\zeta|^{n-1} - \chi|\dot{v}|^{n}$$

where $\beta > |\chi|$, $n \geq 1$, $\beta$ and $\chi$ are parameters describing the shape and amplitude of the hysteresis, and $\nu$ governs the smoothness of the transition from initial slope to the slope of the asymptote. $f(\zeta, \dot{v}) \in \mathbb{R}^1$ is a function with two inputs $\zeta \in \mathbb{R}^1$ and $\dot{v} \in \mathbb{R}^1$ defined as

$$f(\zeta, \dot{v}) = 1 - \text{sign}(\dot{v})|\zeta|^{n-1} - \chi|\dot{v}|^{n}$$

Such a hysteresis and the auxiliary variable $\zeta$ are shown in Figure 1 and Figure 2, respectively, where the parameters are set as $\mu_1 = 3, \mu_2 = 5, \beta = 1, \chi = 0.5, n = 2, \zeta(t_0) = 0, \nu(t) = 4\sin(2t)$.

Remark 1: It can be shown that the hysteresis model $\frac{dx}{dt} = \alpha |\frac{dx}{dt}|(c-u) + B_1 \frac{dx}{dt}$ presented in [14], [15] is a special case of the hysteresis model (1) by setting $\mu_1 = c, \mu_2 = B_1 - c, \beta = \mu, \chi = 0, n = 1$.

**Lemma 1:** Consider the nonlinear dynamic system (2). For any piecewise continuous signal $v$ and $\dot{v}$, the output $\zeta(t)$ is bounded with a bound given as

$$|\zeta(t)| \leq \sqrt{1/|\beta + \chi|}$$

Proof: From dynamic system (2), we have

$$\dot{\zeta} = \dot{v}f(\zeta, \dot{v}) = \dot{v} \left(1 - \left(\text{sign}(\zeta)\text{sign}(\dot{v})\beta + \chi\right)|\zeta|^n\right)$$

Consider a Lyapunov function $V = \frac{1}{2}\zeta^2$. By using the Gronwall’s lemma, we can show that $V(t) \leq V(t_0)$ for all $t \geq t_0$.

**Lemma 2:** For the hysteresis nonlinearity defined in (1)-(2), $\text{sign}(\dot{u}) = \text{sign}(\mu_1 \dot{v})$.

Proof: Equation (2) can be rewritten as

$$\dot{\zeta} = \dot{v} \left(1 - \left(\text{sign}(\zeta)\text{sign}(\dot{v})\beta + \chi\right)|\zeta|^n\right)$$

Note that the term $\left(1 - \left(\text{sign}(\zeta)\text{sign}(\dot{v})\beta + \chi\right)|\zeta|^n\right)$ is bounded for all $t \geq t_0$. Figure 2 shows that $\zeta$ is bounded by $\zeta_M$ when $\zeta(t_0) = 0$.

Taking the derivates of (1) gives that

$$\dot{\mu} = \mu_1 \dot{v} + \mu_2 \dot{\zeta}$$

Since $\text{sign}(\mu_1) = \text{sign}(\mu_2)$, then (10) and the properties (7)-(9) give that

$$\mu_1 \dot{v} > 0 \Rightarrow \dot{\mu} > 0$$

$$\mu_1 \dot{v} < 0 \Rightarrow \dot{\mu} < 0$$

$$\mu_1 \dot{v} = 0 \Rightarrow \dot{\mu} = 0$$
III. COMPENSATION OF HYSTERESIS NONLINEARITY

The best way of compensating for hysteresis effect is to employ a hysteresis inverse to cancel it. A new inverse for the hysteresis in (1)-(2) is proposed as follows

\[ v = HI(u) = \frac{1}{\mu_1} u - \frac{\mu_2}{\mu_1} \zeta_1 \]  
\[ (14) \]

where

\[ \dot{\zeta}_1 = \frac{\dot{u}}{\mu_1 + \mu_2 f(\zeta_1, \frac{u}{\mu_1})} f(\zeta_1, \frac{\dot{u}}{\mu_1}), \quad \zeta_1(t_0) = 0 \]  
\[ (15) \]

\[ \dot{v} = \frac{\dot{u}}{\mu_1 + \mu_2 f(\zeta_1, \frac{u}{\mu_1})} \]  
\[ (16) \]

and the function \( f(., .) \) is defined in (3). Such an inverse is shown in Figure 3. The following results can be obtained.

**Lemma 3:** Consider the nonlinear dynamic system (15). For any piecewise continuous signal \( u \) and \( \dot{u} \), we have the following results.

- The output \( \zeta_1(t) \) is bounded.
- The bound of \( \zeta_1(t) \) is given by \( |\zeta_1(t)| \leq \sqrt{1/(\beta + \gamma)} \).
- For \( \zeta_1(t_0) = 0 \), \( \text{sign}(\dot{\zeta}_1) = \text{sign}(\dot{u}) = \text{sign}(\frac{\dot{u}}{\mu_1}) \).

**Proof:** The first item can be obtained by following similar analysis to the proof of Lemma 1. For \( \zeta_1(t_0) = 0 \), the bound of \( \zeta_1 \) is given by \( |\zeta_1| \leq \sqrt{\frac{1}{\beta + \gamma}} \). It implies that \( f(\zeta_1, \frac{u}{\mu_1}) \geq 0 \) and \( 1 + \frac{\mu_2}{\mu_1} f(\zeta_1, \frac{u}{\mu_1}) > 0 \) since \( \frac{\mu_2}{\mu_1} > 0 \). Then (15) can be rewritten as

\[ \dot{\zeta}_1 \left( 1 + \frac{\mu_2}{\mu_1} f(\zeta_1, \frac{\dot{u}}{\mu_1}) \right) = \frac{\dot{u}}{\mu_1} f(\zeta_1, \frac{\dot{u}}{\mu_1}) \]  
\[ (17) \]

Using (17) and similar analysis in Lemma 2, it can be shown that \( \frac{\dot{u}}{\mu_1} > 0 \) \( \Rightarrow \dot{\zeta}_1 > 0 \), \( \frac{\dot{u}}{\mu_1} < 0 \) \( \Rightarrow \dot{\zeta}_1 < 0 \), and \( \frac{\dot{u}}{\mu_1} = 0 \) \( \Rightarrow \dot{\zeta}_1 = 0 \). Then from (16), the following results can be obtained.

\[ \frac{\dot{u}}{\mu_1} > 0 \Rightarrow \dot{\zeta}_1 > 0, \quad \dot{v} > 0 \]  
\[ (18) \]

\[ \frac{\dot{u}}{\mu_1} < 0 \Rightarrow \dot{\zeta}_1 < 0, \quad \dot{v} < 0 \]  
\[ (19) \]

\[ \frac{\dot{u}}{\mu_1} = 0 \Rightarrow \dot{\zeta}_1 = 0, \quad \dot{v} = 0 \]  
\[ (20) \]

The compensation scheme shown in Figure 4 is described by

\[ v(t) = HI(u_d(t)) = \frac{1}{\mu_1} u_d - \frac{\mu_2}{\mu_1} \zeta_1 \]  
\[ (21) \]

\[ \dot{v} = \frac{\dot{u}_d}{\mu_1 + \mu_2 f(\zeta_1, \frac{u_d}{\mu_1})} \]  
\[ (22) \]

\[ \dot{\zeta}_1 = \frac{\dot{u}_d}{\mu_1 + \mu_2 f(\zeta_1, \frac{u_d}{\mu_1})} f(\zeta_1, \frac{\dot{u}_d}{\mu_1}), \quad \zeta_1(t_0) = 0 \]  
\[ (23) \]

where \( u_d \) is a designed control signal from a feedback law. With the application of hysteresis inverse (21), the following result

\[ \begin{align*}
& u(t_0) = H(HI(u_d(t_0))) = u_d(t_0) \quad \Rightarrow \\
& u(t) = H(HI(u_d(t))) = u_d(t) \quad \forall t \geq t_0
\end{align*} \]
\[ (24) \]

**Proof:** The actual input to the plant is

\[ u(t) = H(HI(u_d)) = \mu_1 v + \mu_2 \zeta \]
\[ = \frac{\mu_1 u_d - \mu_2 \zeta_1}{\mu_1} + \mu_2 \zeta = u_d(t) + \mu_2 (\zeta - \zeta_1) \]  
\[ (25) \]

Then

\[ u(t) - u_d(t) = \mu_2 (\zeta - \zeta_1) \]  
\[ (26) \]

We now prove \( \zeta = \zeta_1 \). From hysteresis inverse (21)-(23) and hysteresis model (1)-(2), we have

\[ \dot{\zeta}_1 = \frac{\dot{u}_d}{\mu_1 + \mu_2 f(\zeta_1, \frac{u_d}{\mu_1})} f(\zeta_1, \frac{\dot{u}_d}{\mu_1}) \]  
\[ (27) \]

\[ \dot{v} = \frac{\dot{u}_d}{\mu_1 + \mu_2 f(\zeta_1, \frac{u_d}{\mu_1})} \]  
\[ (28) \]

\[ \dot{\zeta}_1 = \frac{\dot{v} f(\zeta_1, \dot{v})}{\mu_1 + \mu_2 f(\zeta_1, \frac{u_d}{\mu_1})} \]  
\[ (29) \]

The following inequality is useful in our proof.

\[ (\|\zeta\|^{n-1} \zeta - \|\zeta_1\|^{n-1} \zeta_1) (\zeta - \zeta_1) \geq (\|\zeta\|^{n} - \|\zeta_1\|^{n})(\zeta - \zeta_1) \]  
\[ (30) \]

The inequality (30) can be obtained from the following analysis. If \( \zeta > \zeta_1 \), the proof of (30) is the same as that of \( \|\zeta\|^{n-1} \zeta - \|\zeta_1\|^{n-1} \zeta_1 \geq (\|\zeta\|^{n} - \|\zeta_1\|^{n})(\zeta - \zeta_1) \). For \( \zeta > \zeta_1 \geq 0 \) and \( 0 \geq \zeta > \zeta_1 \), it can be easily obtained that \( \|\zeta\|^{n-1} \zeta - \|\zeta_1\|^{n-1} \zeta_1 \geq (\|\zeta\|^{n} - \|\zeta_1\|^{n})(\zeta - \zeta_1) \). For \( \zeta > 0 > \zeta_1 \), \( \|\zeta\|^{n-1} \zeta - \|\zeta_1\|^{n-1} \zeta_1 \) is \( (\|\zeta\|^{n} + \|\zeta_1\|^{n}) > (\|\zeta\|^{n} - \|\zeta_1\|^{n}) \). For the case \( \zeta \leq \zeta_1 \), the inequality can also be obtained by using the similar analysis.
A Lyapunov function is chosen as $V_\zeta = \frac{1}{2}(\zeta - \zeta_1)^2$. Then its derivative is obtained as
\[
\dot{V}_\zeta = \dot{v} \left( f(\zeta, \dot{v}) - f\left(\zeta_1, \frac{\dot{u}_d}{\mu_1}\right)\right)(\zeta - \zeta_1)
\]
\[
= \dot{v}\left( \text{sign}\frac{\dot{u}_d}{\mu_1} \beta|\zeta_1|^{n-1}\zeta - \text{sign}(\dot{v})\beta|\zeta_1|^{n-1}\zeta
+ \chi|\zeta_1|^n - \chi|\zeta|^n\right)(\zeta - \zeta_1).
\] (31)

There are three cases given as 1) $\frac{\dot{u}_d}{\mu_1} > 0$, 2) $\frac{\dot{u}_d}{\mu_1} < 0$, and 3) $\frac{\dot{u}_d}{\mu_1} = 0$.

1) $\frac{\dot{u}_d}{\mu_1} > 0$ implies that $\dot{v} > 0$ based on (18) in Lemma 3. Then from (31)
\[
\dot{V}_\zeta \leq |\dot{v}|\beta\left(|\zeta_1|^{n-1}\zeta - |\zeta_1|^{n-1}\zeta\right)(\zeta - \zeta_1)
+ |(\zeta_1|^n - |\zeta|^n)(\zeta - \zeta_1)| \leq 0
\] (32)

using $\beta > |\chi|$ and (30).

2) $\frac{\dot{u}_d}{\mu_1} < 0$ implies that $\dot{v} < 0$ from (19) in Lemma 3. Then (31) becomes
\[
\dot{V}_\zeta \leq |\dot{v}|\beta\left(|\zeta_1|^{n-1}\zeta - |\zeta_1|^{n-1}\zeta\right)(\zeta - \zeta_1)
+ |(\zeta_1|^n - |\zeta|^n)(\zeta - \zeta_1)| \leq 0
\] (33)

3) $\frac{\dot{u}_d}{\mu_1} = 0$ implies that $\dot{v} = 0$ from (20) in Lemma 3. Then $\dot{V}_\zeta = 0$.

Thus $\dot{V}_\zeta \leq 0$ for all cases. From Theorem 4.1 in [24], we conclude that $\lim_{t \to \infty}[\zeta(t) - \zeta_1(t)] = 0$. On the other hand, $H\left(H_1(u_d(t_0))\right) = u_d(t_0)$ implies that $\zeta_1(t_0) = \zeta(t_0)$. Therefore we have
\[
V_\zeta(t) = \frac{1}{2}(\zeta(t) - \zeta_1(t))^2 \leq V_\zeta(t_0) = 0 \Rightarrow \zeta(t) - \zeta_1(t) = 0, \forall t \geq t_0
\] (34)

Thus it is obtained that $u(t) = u_d(t) \forall t \geq t_0$ if $u(t_0) = u_d(t_0), \zeta_1(t_0) = \zeta(t_0)$.

IV. MODELLING AND ADAPTIVE CONTROL

A. Model

Typical example of control systems with input hysteresis is magnetic suspension system. An oversimplified schematic representation of a magnetic suspension system is shown in Figure 5. The position of an iron ball is detected by a light source $L$ and a photocell $P$ and compared with a desired reference $r$. The error signal $y - r$ is sent to a controller which generates the control signal-the magnet current $I$. The magnetic force acting upon the iron ball is a nonlinear function of the ball position $y$ and the magnetic flux $\phi$, which is characterized by hysteresis nonlinearity. To cast the system, we consider the current $I$ as the signal $v(t)$ and the magnetic force $F$ acting upon the iron ball as the signal $u(t)$. The hold the ball at some desired position $y = r$ the required force is $u_s$. The amount of the current $v$ needed to generate this force depends on the operating point on the hysteresis characteristic. The input-output model of the magnetic suspension system from the current $I$ to the ball position $y$ can be represented by

The block diagram in Figure 6, where under certain simplifying assumptions the transfer function $G(s) = \frac{k}{(s-p_1)(s-p_2)}$ has two poles $p_1$ and $p_2$. One of the poles $p_1$ is necessary unstable, because the magnetic force decays with the distance and the gravitation force is constant. The dynamics of a magnetic suspension system can be represented by the following second-order uncertain nonlinear system
\[
\dot{y} - (p_1 + p_2)y + p_1p_2y = u(v),
\] (35)

where $p_1$ and $p_2$ are unknown constants, $u(v)$ represents the hysteresis nonlinearity which is described by a Bouc-Wen model in (1). The objective is to design a controller for $v(t)$ to control the ball position $y$ at the desired position $y_r$.

B. Adaptive Backstepping Control

We re-write plant equation (35) as
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u(v) + (p_1 + p_2)x_2 - p_1p_2x_1
\end{align*}
\] (36)

where $x_1 = y$, $x_2 = \dot{y}$, $\theta = [(p_1 + p_2), p_1p_2]^T$, $\psi(x) = [x_2, -x_1]$, $x = [x_1, x_2]^T$. The controller design is achieved by using the recursive backstepping technique and is summarized in Table I, where $c_i, i = 1, 2$ are positive design parameters, $\hat{\theta}$ is the estimate of $\theta$. Then results on the close loop system are given in the following theorem.

Theorem 1: Consider the closed-loop nonlinear system consisting of the plant (35), the hysteresis inverse (40)-(42), the controller (43), and the parameter estimator (45). The system is stable in the sense that all signals in the closed loop system are globally uniformly bounded. Furthermore

- The asymptotic tracking performance is achieved, i.e.,
\[
\lim_{t \to \infty}[y(t) - y_r(t)] = 0
\] (46)

- The transient tracking error performance is given by
\[
\|y(t) - y_r(t)\|_2 \leq \frac{1}{\sqrt{c_1}}(\frac{1}{2}\hat{\theta}(0)^T \Gamma^{-1} \hat{\theta}(0))^{1/2}
\] (47)
TABLE I
ADAPTIVE BACKSTEPPING CONTROL

<table>
<thead>
<tr>
<th>Change of coordinates:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_1 = x_1 - y_r ) \hspace{1cm} (38)</td>
</tr>
<tr>
<td>( z_2 = x_2 - \tilde{y}_r - \alpha_1 ) \hspace{1cm} (39)</td>
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</tbody>
</table>

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<tr>
<th>Inverse:</th>
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<tbody>
<tr>
<td>( v(t) = HI(u_d(t)) = \frac{1}{\mu_1} u_d - \frac{\mu_2}{\mu_1} \zeta_1 ) \hspace{1cm} (40)</td>
</tr>
<tr>
<td>( \dot{v} = \frac{u_d}{\mu_1 + \mu_2 f(\zeta_1, \frac{u_d}{\mu_1})} ) \hspace{1cm} (41)</td>
</tr>
<tr>
<td>( \dot{\zeta}_1 = \frac{u_d}{\mu_1 + \mu_2 f(\zeta_1, \frac{u_d}{\mu_1})} f(\zeta_1, \frac{u_d}{\mu_1}), \zeta_1(t_0) = 0 ) \hspace{1cm} (42)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Adaptive Control Law:</th>
</tr>
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<tbody>
<tr>
<td>( u_d = -c_2 z_2 - z_1 - \bar{\theta}^T \psi(x) + \tilde{y}_r + \alpha_1 ) \hspace{1cm} (43)</td>
</tr>
<tr>
<td>( \alpha_1 = -c_1 z_1 ) \hspace{1cm} (44)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Update Laws:</th>
</tr>
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<tbody>
<tr>
<td>( \dot{\bar{\theta}} = \Gamma \psi(x) z_2 ) \hspace{1cm} (45)</td>
</tr>
</tbody>
</table>

with \( z_i(0) = 0, i = 1, 2 \).

- The transient velocity tracking error performance is given by

\[ \| \dot{y} - \dot{y}_r \|_2 \leq \left( \frac{1}{\sqrt{c_2}} + \sqrt{c_1} \right) \left( \frac{1}{2} \bar{\theta}^T(0) \Gamma^{-1} \dot{\theta}(0) \right)^{1/2} \] \hspace{1cm} (48)

**Proof:** We define a positive Lyapunov function as

\[ V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \ \frac{1}{2} \bar{\theta}^T \Gamma^{-1} \dot{\theta} \] \hspace{1cm} (49)

where \( \dot{\theta} = \theta - \hat{\theta} \). Then the derivative of \( V \) along with the controller (43) and the parameter estimator (45) is given by

\[ \dot{V} = z_1 \dot{z}_1 + z_2 \dot{z}_2 + \bar{\theta}^T \Gamma^{-1} \dot{\theta} \]

\[ = -c_1 z_1^2 - c_2 z_2^2 + \bar{\theta}^T \Gamma^{-1} (\Gamma \psi z_2 - \dot{\theta}) \]

\[ = -c_1 z_1^2 - c_2 z_2^2 \] \hspace{1cm} (50)

(50) shows that \( V(t) \) is globally uniformly bounded. This implies that \( z_1, z_2, \dot{\theta} \) are bounded. The state variables \( x_1, x_2 \) and the parameter estimates \( \hat{\theta} \) are also bounded. Thus \( u_d \) is bounded from (43) because of the boundedness of \( z_1, z_2, \dot{\theta} \).

By applying the LaSalle-Yoshizawa theorem in [25] to (50), it further follows that \( z_i(t) \rightarrow 0, i = 1, 2 \) as \( t \rightarrow \infty \), which implies that \( \lim_{t \rightarrow \infty} |x(t) - y_r(t)| = 0 \).

Since \( V \) is non increasing from (50), we have

\[ \| z_1 \|_2 \leq \int_0^\infty |z_1(\tau)|^2 d\tau \leq \frac{1}{c_1} V(0) \] \hspace{1cm} (51)

Thus, by setting \( z_1(0) = z_2(0) = 0 \), we obtain

\[ V(0) = \frac{1}{2} \bar{\theta}^T(0) \Gamma^{-1} \dot{\theta}(0) \] \hspace{1cm} (52)

a decreasing function of \( \Gamma \), independent of \( c_1 \). This means that the bounds resulting from (51) and (52)

\[ \| z_1 \|_2 \leq \frac{1}{\sqrt{c_1}} \left( \frac{1}{2} \bar{\theta}^T(0) \Gamma^{-1} \dot{\theta}(0) \right)^{1/2} \] \hspace{1cm} (53)

can be asymptotically reduced either by increasing \( c_1 \) or by simultaneously increasing \( \Gamma \). The bound for \( z_1 \) is explicit. We get

\[ \| \dot{y} - \dot{y}_r \|_2 \leq \| z_2 - c_1 z_1 \|_2 \leq \| z_2 \|_2 + c_1 \| z_1 \|_2 \] \hspace{1cm} (54)

Similarly, we can get \( \| z_2 \|_2 \leq \frac{1}{\sqrt{c_2}} \sqrt{V(0)} \). Along with (53) we get

\[ \| \dot{y} - \dot{y}_r \|_2 \leq \left( \frac{1}{\sqrt{c_2}} + \sqrt{c_1} \right) \left( \frac{1}{2} \bar{\theta}^T(0) \Gamma^{-1} \dot{\theta}(0) \right)^{1/2} \] \hspace{1cm} (55)

**V. SIMULATION STUDY**

In this section we test our proposed backstepping controllers on the magnetic suspension model (35). For simulation studies, the following values are selected as “true” parameters for the system \( \mu_1 = 0.8, \mu_2 = -1 \). The parameters in the Bouc-Wen hysteresis are given as \( \mu_1 = 1, \mu_2 = 1, \beta = 1, \chi = 0.5, n = 2 \). The design objective is to hold the ball at a desired position \( y_r = 1.2 \sin(0.5\pi t) \). We take the following set of design parameters: \( \Gamma = I_2, \ c_1 = 2, c_2 = 5 \). The initials are set as \( y(0) = 0.6, \ y_r(0) = 1.8 \), and \( \theta(0) = [0, -1] \), respectively.

The simulation results with the controller designed by our proposed scheme and the scheme in [15], where the effect of hysteresis is considered as a disturbance, presented in the Figures 7-10 show that the tracking error and the control input \( v(t) \) is remarkable that the tracking performance is improved greatly by our proposed scheme. Clearly, the simulation results verify our theoretical findings and show the effectiveness of our proposed scheme.

**Fig. 7.** Tracking error \( y - y_r \).

**VI. CONCLUSION**

This paper presents a feedback backstepping adaptive controller design for a magnetic suspension system preceded by the hysteresis actuator nonlinearity. We propose a new hysteresis inverse to compensate the effect of the Bouc-Wen hysteresis. The inverse function is employed in the backstepping controller design. For the design and implementation
we also give an explicit bound on the asymptotically converges to zero. Besides showing stability, loop systems are bounded and that the output tracking error in terms of design parameters. It is shown that all signals in the closed-loop system parameters. It is shown that all signals in the closed-loop systems are bounded and that the output tracking error in terms of design parameters. Simulation results illustrate the effectiveness of our proposed scheme.

REFERENCES


