Multi-surface Sliding Control of MIMO Autonomous Flight Systems

Michael Norton, Suiyang Khoo, Abbas Kouzani and Alex Stojcevski
School of Electrical Engineering
Deakin University
Waurn Ponds, Victoria, Australia 3022
Email: mjno@deakin.edu.au

Abstract—In this paper, a multi-surface sliding control (MSSC) is proposed for trajectory tracking of 6 degrees of freedom (6-DOF) inertia coupled aerial vehicles with multiple inputs and multiple outputs (MIMO). It is shown that an iterative MSSC design can be carried out to control flight. Using MSSC on MIMO autonomous flight systems creates confluent control that can account for model mismatches, system uncertainties, system disturbances and excitation in internal dynamics. We prove that the MSSC system guarantees asymptotic output tracking and ultimate uniform boundedness of the system. Simulation results are presented to validate the analysis.

I. INTRODUCTION

Over the past decades, control scientists have been striving to develop robust control systems for aerial vehicles, so that they can maintain steady flight in all types of conditions. UAVs in civil, military and commercial markets have focused on systems with greater emphasis on development of robust and fault-tolerant controls [1]–[4]. As flight systems have 6 degrees of freedom (6-DOF) and 12 states, meaning they have three translational outputs (x,y,z) and three rotational outputs (roll, pitch and yaw) making 6-DOF and 12 state outputs but have only four control inputs \(d_a, d_r, d_e\) and \(d_t\), the system is shown to be under-actuated with the control inputs shared between the translational and the rotational systems. This makes the control of flight systems extremely difficult. Some of the greatest hurdles faced by control engineers of aerial systems both small and large, are the control of coupled rotational and translational dynamics and maintenance of stable trajectory tracking in hazardous environments with system uncertainties. For many years, systems had been designed with separation of the longitudinal and lateral dynamics, ignoring the inertia coupling completely, causing conflicts between the two systems. During flight, the longitudinal controller is unaware of the lateral controller’s movement, leaving the two controllers locked in a fight for supremacy, with the overall system losing. Flight systems were also linearized which meant that they could only give an approximation of the system states. This meant that systems might became unstable with the introduction of uncertainties.

Recently, several nonlinear control techniques have been proposed for aerial flight, such as feedback linearization, nonlinear dynamic inversion [5]–[7], sliding mode control [8], [9] and adaptive control techniques utilizing back-stepping and super-twisting methods [10], [11].

In this paper, we show a design of a new MSSC control of UAV systems, to deal with the inertia coupling and trajectory tracking for 6-DOF flight. The new MSSC has been achieved by designing a new simple transformation [12], freeing the under-actuated controls allowing the MSSC to control the system. The use of MSSC [13], [14] and high order sliding mode control [15] for control of under-actuated nonlinear systems [16] with mismatched uncertainties has been proven as a robust and desirable control methodology for single input and single output (SISO) systems, as in [1], [13], [17]–[19]. The proposed novel control scheme creates confluent control of UAV systems. This paper addresses the asymptotic control problem of 6-DOF inertia coupled control UAV systems, maintaining stability whilst tracking a desired trajectory.

II. 6-DOF INERTIA COUPLED ATMOSPHERIC FLIGHT DYNAMICS

An Earth fixed reference frame will be considered for expressing the equations of flight, with positive \(x\) pointing due north, positive \(y\) pointing due east and positive \(z\) pointing towards the center of the Earth. The origin is a fixed point on the surface of the Earths surface which can be referenced. The body frame of the aerial vehicle is expressed as, positive \(u\) pointing along the fuselage, positive \(v\) pointing 90\(^\circ\) from the \(u\) plane along the right wing, and positive \(w\) pointing perpendicular to plane \(u\) and \(v\) and downwards. The standard Euler angles \(\phi, \theta, \psi\) and rotational body rates \(P, Q, R\) refer to the rolling, pitching and yawing, angles and rates, respectively. Some symbols used in this paper are defined in Table 1. The planes of reference can be seen in figures 1 and 2.

We can express our governing equations and coefficients of atmospheric flight as follows based on [2], [20]–[22] (A shorthand notation is used to write the state equations in which \(s_\theta = \sin \theta, c_\psi = \cos \psi\), etc.) :
TABLE I
DEFINITION OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$, $y$, $z$</td>
<td>earth axes</td>
</tr>
<tr>
<td>$u$, $v$, $w$</td>
<td>linear velocity components along body axes</td>
</tr>
<tr>
<td>$\phi$, $\theta$, $\psi$</td>
<td>euler angles, roll, pitch and yaw</td>
</tr>
<tr>
<td>$P$, $Q$, $R$, $\beta$</td>
<td>angular velocity components along body axes</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle of attack</td>
</tr>
<tr>
<td>$I_{xx}$, $I_{yy}$, $I_{zz}$</td>
<td>moments of inertia about body axes</td>
</tr>
<tr>
<td>$dt$, $da$, $dr$, $de$</td>
<td>thrust, aileron, rudder, elevator coefficients</td>
</tr>
<tr>
<td>$m$</td>
<td>mass of the aircraft</td>
</tr>
<tr>
<td>$q$</td>
<td>air pressure</td>
</tr>
<tr>
<td>$V$</td>
<td>total velocity of the aircraft</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>$T_{max}$</td>
<td>thrust coefficient</td>
</tr>
</tbody>
</table>

Fig. 1: Aerial vehicle earth frame plane of reference

Fig. 2: Aerial vehicle body frame plane of reference

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} = \begin{bmatrix}
c_{\theta}c_{\phi} & s_{\phi}c_{\theta} - c_{\theta}s_{\phi}c_{\psi} & c_{\phi}s_{\theta}c_{\psi} - s_{\phi}c_{\theta} \\
-c_{\phi} & s_{\phi}c_{\theta} + c_{\theta}s_{\phi}c_{\psi} & c_{\phi}s_{\theta}c_{\psi} + s_{\phi}c_{\theta} \\
-s_{\theta} & s_{\phi}s_{\theta}c_{\psi} + c_{\phi}c_{\theta} & c_{\phi}s_{\theta}s_{\psi} - s_{\phi}c_{\theta}
\end{bmatrix} \begin{bmatrix}
u \\
v \\
w
\end{bmatrix},
\]

\[
\dot{\mathbf{v}} = f_2(P, Q, R, \theta, \psi, \alpha, \beta) + f_4(P, Q, R, \theta, \psi, \alpha, \beta),
\]

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
s_\phi \tan \theta & c_\phi \tan \theta & P \\
c_\phi & -s_\phi & Q \\
s_\phi \sec \theta & c_\phi \sec \theta & R
\end{bmatrix} + f_3(P, Q, R, \theta, \psi, \alpha, \beta),
\]

\[
\dot{\mathbf{P}} = f_4(P, Q, R, \theta, \psi, \alpha, \beta) + f_6(P, Q, R, \theta, \psi, \alpha, \beta),
\]

where,

\[
f_2(P, Q, R, \theta, \psi, \alpha, \beta) = f_{2a}(P, Q, R) \begin{bmatrix} u \\ v \\ w \end{bmatrix} + f_{2b}(\theta, \psi),
\]

\[
f_{2a}(P, Q, R) = \begin{bmatrix} 0 & R & -Q \\ -R & 0 & P \\ Q & -P & 0 \end{bmatrix},
\]

\[
f_{2b}(\theta, \psi) = \begin{bmatrix} -g \sin \theta \\ g \cos \theta \sin \psi \\ g \cos \theta \cos \psi \end{bmatrix},
\]

\[
f_{2c}(P, Q, R, \alpha, \beta) = \frac{qSClI_{xx} + qSbI_{zz} - \frac{m}{l_{yy}}(C_{y\beta} + C_{z\beta})}{qS(C_{L} + C_{z\alpha} + \frac{m}{l_{yy}}(C_{y\alpha} + C_{z\beta}))},
\]

\[
f_{4a}(P, Q, R) = \frac{(PQQ + I_{yy} QR + I_{xx} PR - I_{yy} R^2) - I_{xx} P^2}{I_{xx} R_{yy} R^2 - I_{xx} P^2},
\]

\[
f_{4b}(\alpha, \beta) = \frac{qS(C_{y\alpha} + C_{z\alpha}) + \frac{m}{l_{yy}}(C_{y\alpha} + C_{z\beta}) + \frac{m}{l_{yy}}(C_{y\alpha} + C_{z\beta})}{qS(C_{y\alpha} + C_{z\alpha} + \frac{m}{l_{yy}}(C_{y\alpha} + C_{z\beta}))},
\]

The inertial constants in (1-4), are given in the appendix.
III. COORDINATE TRANSFORMATION

We can rewrite equations (1)-(4), as the following differential equations:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_3) x_2, \\
\dot{x}_2 &= f_2(x_4, \theta, \psi, \alpha, \beta) + b_1 u(t) + b_1 u_2(t), \\
\dot{x}_3 &= f_3(x_4), \\
\dot{x}_4 &= f_4(x_4, \alpha, \beta) + b_2 u(t),
\end{align*}
\]

where

\[
\begin{align*}
x_1 &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}, & x_2 &= \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \\
x_3 &= \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, & x_4 &= \begin{bmatrix} P \\ Q \\ R \end{bmatrix}, \\
f_1(x_3) &= \begin{bmatrix} c_\phi c_\psi & s_\phi s_\psi & -s_\phi c_\psi & c_\phi s_\psi + s_\phi s_\psi \\
c_\phi s_\psi & s_\phi c_\psi & c_\phi s_\psi & c_\phi s_\psi - s_\phi c_\psi \\
-s_\phi & s_\phi c_\theta & c_\phi & s_\phi c_\theta \\
0 & c_\theta & 0 & c_\theta \end{bmatrix}, \\
f_3(x_3) &= \begin{bmatrix} 1 & s_\phi \tan \theta & c_\phi \tan \theta \\
0 & c_\phi & -s_\phi \\
0 & s_\phi \sec \theta & c_\phi \sec \theta \end{bmatrix}, \\
b_1 &= \frac{1}{m} \begin{bmatrix} 0 & 0 & 0 \\
0 & qSC_{ydr} & 0 \\
0 & 0 & qSC_{zde} \end{bmatrix}, \\
b_2 &= \begin{bmatrix} qSbC_{lda} & qSbC_{ldr} & 0 \\
0 & 0 & qSC_{mdr} \\
T_{max} & 0 & 0 \end{bmatrix}, \\
b_t &= \begin{bmatrix} d_a \\
d_r \\
d_e \end{bmatrix}, & u_1(t) &= \begin{bmatrix} d_t \\ 0 \\ 0 \end{bmatrix}.
\end{align*}
\]

It is worth mentioning that the control input \( u(t) \) is shared by subsystems 2 and 4. Therefore system (5) is an under-actuated system where \( u, u_t \) are \( 3 \times 1 \) control vectors, \( f_1(\cdot), f_2(\cdot), b_1, b_2, b_t \) are \( 3 \times 3 \) matrices with each element formed by either constant or smooth function and \( f_3(\cdot), f_4(\cdot) \) are \( 3 \times 1 \) vectors with each element formed by a continuous function. To eliminate the under-actuated effect, we propose the following simple transformation where we define new variables \( z_1, z_2, z_3, z_4 \) as

\[
\begin{align*}
z_1 &= x_1, \\
z_2 &= x_2 - b_1 b_2^{-1} x_4, \\
z_3 &= x_3, \\
z_4 &= x_4.
\end{align*}
\]

Differentiating (6)-(9) with respect to time and using (5), we have

\[
\begin{align*}
\dot{z}_1 &= f_1(z_3) x_2, \\
\dot{z}_2 &= f_2(z_4, \theta, \psi, \alpha, \beta) + b_1 u \frac{d}{dt} b_2^{-1} f_4(z_4, \alpha, \beta) - \frac{d}{dt} b_2^{-1} f_4(z_4, \alpha, \beta) z_4, \\
\dot{z}_3 &= f_3(z_3) z_4, \\
\dot{z}_4 &= f_4(z_4, \alpha, \beta) + b_2 u(t). 
\end{align*}
\]

We can further rewrite equation (10) into the cascaded form:

\[
\begin{align*}
\dot{z}_1 &= z_2 + d_1, \\
\dot{z}_2 &= z_3 + b_1 u + d_2, \\
\dot{z}_3 &= f_3(z_3) z_4, \\
\dot{z}_4 &= f_4(z_4, \alpha, \beta) + b_2 u(t),
\end{align*}
\]

where

\[
\begin{align*}
d_1 &= -z_2 + f_1(z_3) x_2, \\
d_2 &= -z_3 + f_2(z_4, \theta, \psi, \alpha, \beta) - \frac{d}{dt} b_2^{-1} f_4(z_4, \alpha, \beta) z_4.
\end{align*}
\]

**Remark 1.** Clearly, after the simple transformation, system (11) is free from under actuated effect. In addition, by (6)-(9), the convergence of \( z_1 \sim z_4 \) to zero also implies the convergence of the original system states, \( x_1 \sim x_4 \), to zero. This means that the change of coordinates given in (6)-(9) is physically meaningful.

For brevity, we assume there exist some bounded mismatched uncertainties at every subsystem of (11) and then system (11) can be re-written as

\[
\begin{align*}
\dot{z}_1 &= z_2 + d_1 + \dot{\delta}_1, \\
\dot{z}_2 &= z_3 + b_1 u + d_2 + \dot{\delta}_2, \\
\dot{z}_3 &= f_3(z_3) z_4 + \dot{\delta}_3, \\
\dot{z}_4 &= f_4(z_4, \alpha, \beta) + b_2 u(t) + \dot{\delta}_4,
\end{align*}
\]

where \( \dot{\delta}_1, \ldots, \dot{\delta}_4 \) represent the bounded uncertainties.

IV. MULTIPLE SLIDING SURFACE CONTROL

Let us consider system (12) and define \( z_{id} = y_d \), where \( y_d \) is the UAV’s desired trajectory. To design a multi-surface sliding controller, we need to define 4 sliding variables

\[
s_i = z_i - z_{id}, \quad i = 1, \ldots, 4,
\]

where \( z_{id} \) is the desired value for \( z_i \). The sliding variables \( s_1, \ldots, s_4 \) are \( 3 \times 1 \) column vectors with \( s_i \) represents the \( i \)th element of the \( i \)th sliding variable (e.g.: \( s_1 \) can be represented as \( s_1 = \begin{bmatrix} s_{11} & s_{12} & s_{13} \end{bmatrix}^T \)).

Differentiating \( s_1 \) with respect to time, we have

\[
\dot{s}_1 = \begin{bmatrix} \dot{z}_1 - \dot{z}_{1d} \\
\dot{z}_2 + \Delta_1 - \dot{z}_{1d} \\
\dot{z}_2 + \Delta_2 - \dot{z}_{2d} \end{bmatrix},
\]

where

\[
\begin{align*}
\dot{z}_1 &= \dot{z}_2 + \dot{\delta}_1, \\
\dot{z}_2 &= \dot{z}_3 + b_1 u + \dot{\delta}_2, \\
\dot{z}_3 &= f_3(z_3) z_4 + \dot{\delta}_3, \\
\dot{z}_4 &= f_4(z_4, \alpha, \beta) + b_2 u(t) + \dot{\delta}_4.
\end{align*}
\]
where $\tilde{\Delta}_1 := d_1 + \delta_1$. Let $z_{2d}$ be a virtual controller and design
\[ z_{2d} = -K_1 s_1 + \dot{z}_{1d}, \]  
(15) 
where $K_1 = \text{diag}(k_1, k_1, k_1)$ is a $3 \times 3$ diagonal matrix with $k_1 > 0$. Substituting (15) into (14) to get
\[ \dot{s}_1 = s_2 - K_1 s_1 + \Delta_1. \]  
(16) 
Similarly, differentiating $s_2$ with respect to time yields
\[ \dot{s}_2 = \bar{\Delta}_2, \]  
(17) 
where $\bar{\Delta}_2 = d_2 + \delta_2$. The virtual controller, $z_{3d}$, can be designed as:
\[ z_{3d} = -K_2 s_2 + \bar{\Delta}_2. \]  
(18) 
where $K_2$ is another $3 \times 3$ diagonal matrix with positive number, $k_2$, as diagonal elements. Substituting (18) into (17), we get
\[ \dot{s}_2 = s_3 - K_2 s_2 + b_k u_t + \bar{\Delta}_2. \]  
(19) 
Since $T_{\text{max}}$ is a positive constant, we can design $u_t$ as
\[ u_t(t) = -\text{diag}\{k_t, 0, 0\} s_2, \]  
(20) 
where $k_t > 0$ is a positive constant.

Next, differentiating $s_3$ with respect to time yields
\[ \dot{s}_3 = \bar{\Delta}_3, \]  
(21) 
where $\bar{\Delta}_3 := \bar{\delta}_3$. The $3^{rd}$ virtual controller, $z_{4d}$, can be designed as
\[ z_{4d} = f_3(z_3)^{-1} (-K_3 s_4 + \bar{\Delta}_3), \]  
(22) 
where $K_3$ is a $3 \times 3$ diagonal matrix with $k_3 > 0$ as the diagonal elements. Substituting (22) into (21) to results in
\[ \dot{s}_3 = f_3(z_3)s_4 - K_3 s_3 + \bar{\Delta}_3. \]  
(23) 
Finally, the time derivative of $s_4$ gives
\[ \dot{s}_4 = \bar{\Delta}_4, \]  
(24) 
where $\bar{\Delta}_4 := \bar{\delta}_4$. The actual controller can be designed as:
\[ u(t) = \begin{cases} b_k^{-1} (-f_4 - K_4 s_4 - \mu_4 \text{sign}(s_4) + \dot{z}_{4d}) & \text{for } |q| > 0 \\ 0 & \text{for } q = 0 \end{cases} \]  
(25) 
where $K_4$ is a $3 \times 3$ diagonal matrix with $k_4 > 0$ as the diagonal elements, $\mu_4 > 1 + \Delta_{\text{max}}$. \text{sign} \left( [a \ b \ c]^T \right) := [\text{sign}(a) \ \text{sign}(b) \ \text{sign}(c)]^T$. Using the control laws (15), (18), (22) and (25), it is easy to obtain
\[ \begin{align*} \dot{s}_1 &= s_2 - K_1 s_1 + \Delta_1, \\
\dot{s}_2 &= s_3 - K_2 s_2 + b_k u_t + \Delta_2, \\
\dot{s}_3 &= f_3(z_3)s_4 - K_3 s_3 + \Delta_3, \\
\dot{s}_4 &= -K_4 s_4 - \mu_4 \text{sign}(s_4) + \Delta_4. \end{align*} \]  
(26)
V. NUMERICAL SIMULATION RESULTS

In order to validate the effectiveness and strength of the proposed method, nonlinear 6-DOF simulation results for an atmospheric flight model are presented. The initial flight conditions $x = 1m$, $y = 0.5m$, $z = 101m$, $u = 40m/s$, $v = 0.03m/s$, $w = 0.01m/s$, $\phi = 0.3$, $\theta = 0.5$, $\psi = 0.9$, $P = 0/s$, $Q = 0/s$, $R = 0/s$ are considered for simulation. The desired trajectory was set as $x_d = 40m/s$, $y_d = 20sin(0.05\pi t)$ and $z_d = 100m$. The controller parameters are selected as $K_1 = diag\{2,2,2\}$, $K_2 = diag\{5,5,5\}$, $K_3 = diag\{30,30,30\}$, $K_4 = diag\{50,50,50\}$, $k_t = 10$ and $m_4 = diag\{10,10,10\}$. The controller design parameters chosen for a Cessna 182T aircraft are given in the appendix.

Two simulation cases are presented here. The first case is the control of a UAV using a MSSC controller without disturbances. The second case is the control of a UAV using a MSSC controller with disturbances. The disturbances are given as:

$$\delta_1 = \begin{bmatrix} 0.5cos(0.5\pi t)(x_1 + x_2 + x_3 + x_4) \\ -0.5cos(0.3\pi t)(x_2^2 + x_2^3 + x_4) \end{bmatrix},$$

$$\delta_2 = [-0.2, 0.1, -0.5]^T,$$

$$\delta_3 = [5, 5, 0.1]^T,$$

$$\delta_4 = [5, 5, 5]^T.$$

Case 1. Control using MSSC without disturbances

The simulation results are shown in Figure 3 and 4. Figure 3 verifies that the aerial vehicle trajectory converges. Figure 4 is the control effort required.

Case 2. Control using MSSC with disturbances

The simulation results are shown in Figure 5 and 6. Figure 5 verifies that the aerial vehicle trajectory converges and the tracking error is bounded. Figure 6 is the control effort required.

VI. CONCLUSION

In this paper, a MSSC for MIMO atmospheric flight systems was presented for trajectory tracking of inertia coupled atmospheric flight vehicles. The proposed control strategy, based on MSSC, and a simple transformation technique, allows for design of a MSSC that can take into account differing system types, bounded unknown system parameters and system disturbances. The presented method was applied to an aerial vehicle with 6-DOF coupled dynamics to control the UAV system. The simulation results show the effectiveness of the proposed control scheme. It is seen that performance of the proposed MSSC gives convergence to a bound.

VII. APPENDIX

[h/t] Inertia Moment Constants:

$$I_1 = \frac{I_{xx} - I_{yy} + I_{zz}}{D},$$

$$I_2 = \frac{I_{yy} - I_{zz} - I_{xx}^2}{D},$$

$$D = (I_{zz}^2 - I_{xx}I_{xx})^{-1},$$

$$I_{xx} = 1285.315kg/m^2,$$

$$I_{yy} = 1824.931kg/m^2,$$

$$I_{zz} = 0kg/m^2,$$

$$I_{zz} = 2666.894kg/m^2.$$

Cessna 182T Flight Coefficients:

$$C_D = 0.031, \quad C_{L_{\beta}} = -0.089,$$

$$C_{x_{\alpha}} = 0.13, \quad C_{l_{\beta}} = -0.47,$$

$$C_{x_{\beta}} = 0.0, \quad C_{l_{\alpha}} = 0.096,$$

$$C_{w_{\beta}} = -0.31, \quad C_{d_{\alpha}} = 0.178,$$

$$C_{w_{\beta}} = -0.337, \quad C_{d_{\alpha}} = 0.0417,$$

$$C_{\psi_{\beta}} = -0.21, \quad C_{m_{\psi}} = 0.0,$$

$$C_{y_{dr}} = 0.187, \quad C_{c_{\alpha}} = -0.89,$$

$$C_{z_{de}} = 0.06, \quad C_{m_{\alpha}} = -5.2,$$

$$C_{z_{\alpha}} = 4.6, \quad C_{m_{\alpha}} = -12.4,$$

$$C_{z_{do}} = 1.7, \quad C_{m_{\alpha}} = 0.178,$$

$$C_{z_{z_{\alpha}}} = 3.0, \quad C_{n_{\alpha}} = 0.065,$$

$$C_{z_{d_{\alpha}}} = 0.43, \quad C_{n_{\alpha}} = -0.03,$$

$$c = 4.9, \quad C_{n_{\alpha}} = -0.099,$$

$$b = 35.8, \quad C_{n_{\alpha}} = -0.053,$$

$$S = 174, \quad C_{n_{\alpha}} = -0.0657,$$

$$m = 2645, \quad T_{max} = 2530.$$
REFERENCES


